



S-Parameter Modelling for 6-Layer PCB Interconnects with Kron-Branin Formalism

Z. Xu, Y. Liu, B. Ravelo and O. Maurice

Normandy University UNIROUEN, ESIGEELEC, IRSEEM, F-76000 Rouen, France {zhifei.xu, yang.liu, blaise.ravelo, olivier.maurice}@esigeelec.fr

Keywords: Kron-Branin (KB) Formalism, Tensorial Analysis of Network (TAN), Multilayer PCB, Interconnects Model.

Mots-Clés: Formalisme de Kron-Branin (KB), Analyse Tensorielle des Réseaux (ATR), Circuit imprimé multicouche, Modèle d'interconnexion.

Abstract

With the increase of the printed circuit board (PCB) speed and design complexity, the electrical interconnect modelling plays more and more important role for the design and manufacturing engineers. More accurate and faster modelling methodology is necessary in particular of the multilayer PCBs. To answer to such requirement, the present paper proposes an uncommon modelling methodology of 6-layer PCB electrical interconnect with tensorial analysis of network (TAN) based on the Kron-Branin (KB) formalism. The KB method feasibility is applied first time, to multilayer PCB structure. The proof-of-concept (POC) is constituted by microstrip lines, lands, pads and via implemented on stacked substrate presenting different relative dielectric 3.3 to 4.3 and thickness $h = 2$ mm. The KB method validity is verified with comparison of S-parameters from commercial tool ADS® circuit simulator. An excellent agreement between the KB model and ADS simulations is discovered.

Résumé

Avec l'augmentation de la vitesse de transmission des données et de la complexité de conception, les effets d'interconnexions électriques jouent un rôle de plus en plus important en ingénierie de fabrication des cartes électroniques. Une méthodologie de modélisation de plus en plus performante en matière de la précision et de la rapidité est nécessaire notamment pour les cartes électroniques multicouches. Pour répondre à cette exigence, cet article propose une méthodologie de modélisation non-conventionnelle appliquée à une carte de six-couches métalliques avec l'Analyse Tensorielle des Réseaux (ATR) à base du formalisme de Kron-Branin (KB). Il s'agit d'une première étude de faisabilité du formalisme de KB pour les structures multicouches. La structure de preuve de faisabilité est constituée des lignes microrubans, des plots métalliques de soudure et des vias empilés et collés sur des substrats diélectriques présentant des permittivités relatives de 3,3 à 4, et d'épaisseur identique 2mm. La validité de la méthode KB est vérifiée avec comparaison des paramètres S simulés avec un outil commercial. Une excellente corrélation avec la simulation a été obtenue.

1 Introduction

The rapid progress of complexity and signal processing speed in high density PCBs [1] was majorly driven by the increase of telecommunication speed. It is accelerated by the human demands in robots, vehicles and other assistant tools. The manufacturers, designers in PCB's field are challenged by this explosive progress. As always, interconnect is the most common element in design and modelling process, it is also the most difficult to integrate and analyse in the model and EDA tools [2]. The most important reason is the electromagnetic field computations are strongly depending on different parameters of electrical, material and also geometric aspects. And interconnect imperfections as via discontinuities [3-4] are one of the most critical issues. Various noises and matching problems, like crosstalk, Simultaneous Switching Noise (SSN), mismatching, are generated unintentionally in the high density multilayer PCBs by the interconnect imperfections [5]. The operation status even the function of the boards will be influenced by these electromagnetic compatibility (EMC) and signal integrity (SI) issues.

With the complexity of interconnect grows, its electrical design and modelling will become more difficult for designers because for most CAD tools, the progress of technology always outpace the development of interconnect design [2]. Despite many tools have been developed to permit deep and fundamental analysis including both electrical and physical aspects, however, none of them is sufficient. Thus, they are calling for a more sophisticated, sufficient and accurate computer-aided design method for system level design. This paper proposes an innovative model of multilayer PCB interconnect with vias, pads and lands with TAN KB formalism. This innovative formalism permits fast computations and high adaptability to the complex system which can overcome the

difficulties from designers and manufactories. The unconventional Branin model was established and developed by Branin in 1960s [6]. It provides a more elegant way to calculate the voltage and current along the transmission line by applying two internal voltage sources at the ends of transmission line. The association between Kron and Branin method was initiated by O. Maurice and his group for the modelling of electronic complex structures constituted by transmission lines (TLs) [7]. The KB method effectiveness for the Y-shape signal distribution interconnects was emphasized recently in [8-9]. In this present paper, the KB method is first time, applied to multilayer PCB interconnect structures.

2 KB modelling methodology of 6-layer PCB structure

2.1 Equivalent topology of the structure

Fig. 1 and Fig. 2 represents a standard 6-layer PCB structure comprised of different permittivity dielectric substrates having same thickness $h_{1,2,3,4,5} = 2$ mm [10]. The posed-problem diakoptic analysis [11] suggests the consideration of elementary cells: vias, pads, lands (radius respectively $r_{\text{via}} = 0.1$ mm, $r_{\text{pads}} = 0.3$ mm, and $r_{\text{lands}} = 0.45$ mm), microstrip lines (physical lengths $d_1 = d_2 = 20$ mm and widths $w_1 = w_2 = 0.2$ mm) and one power and ground plane.

The diakoptic analysis proceeds in two steps. Step 1 is the elementary problem splitted structure into two parts essentially constituted by via cells (via holes, lands and pads) and TLs assumed with characteristic impedances $Z_{1,2}$ and electrical lengths $\theta_{1,2}$. Step 2 is the combination of the elementary structure theorization with graph topology as depicted in Fig. 3. At higher frequencies, the impacts of inductances, capacitance, and TLs are significantly influencing the structure electrical responses. So, the equivalent TAN is essentially constituted by Branin's model in mesh ($\Sigma_{1,2}$) with the input R_1 -resistive source V_1 in mesh (Σ_1), via LC-ladders with meshes ($\Sigma_{3,...,7}$) and another Branin's model in mesh ($\Sigma_{8,9}$) with R_2 -resistive output source V_2 in mesh (Σ_9). Then, the outstanding mathematical abstraction of the 6-layer PCB problem can be formulated intuitively with KB characteristic matrix.

The branches crossed by E_1 and E_2 are coupled by the Branin's coupling voltage source which is constituted by the characteristic impedance Z_c and the propagation constant [12] calculated with (1) and (2):

$$\theta(j\omega) = [\alpha(\omega) + j\beta(\omega)]d \quad (1)$$

$$Z_c = \frac{60}{\sqrt{\epsilon_{\text{eff}}}} \ln \left(\frac{5.98h}{0.8w+t} \right) \quad (2)$$

where ω is the angular frequency, $\alpha(\omega)$ is the attenuation constant and

$$\beta(j\omega) = \omega \sqrt{\epsilon_{\text{eff}}} / c \quad (3)$$

is the phase constant by taking c as the vacuum light speed and ϵ_{eff} the effective permittivity. Then, the branches crossed by E_3 and E_4 are similarly coupled as E_1 and E_2 .

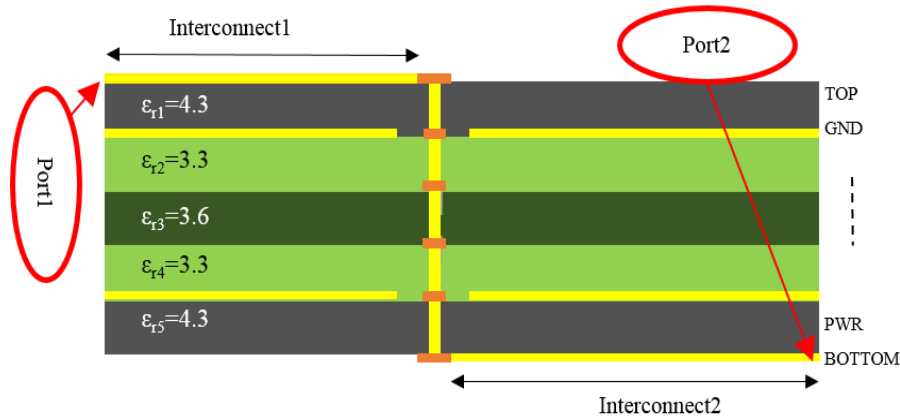


Figure 1: 6-layer PCB structure under study cross section [10].

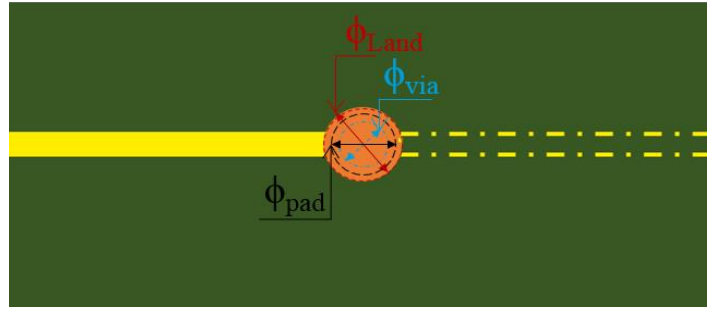


Figure 2: 6-layer PCB structure under study top view.

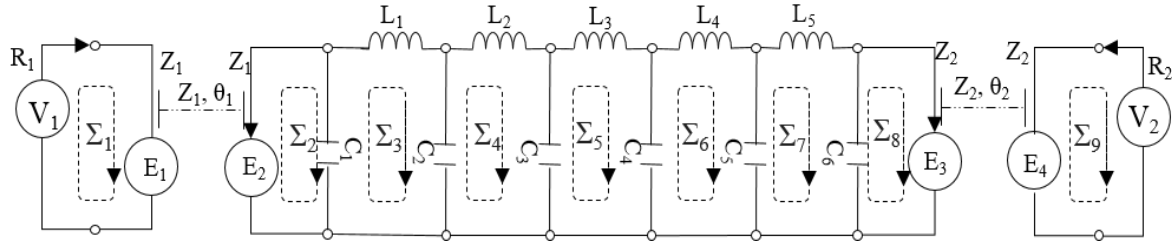


Figure 3: KB model equivalent topology of the structure introduced in Fig. 1.

2.2 KB matrix of the graph topology

The impedance matrix (4) of this structure can be derived from Fig. 3 with KB method, it permits us to compute the current along this multilayer structure, all elements constituted in this matrix will be described in section 3.

$$[Z] = \begin{bmatrix} Z_1 + R_1 & (Z_{C1} - Z_1)e^{-\theta_1} & -Z_{C1}e^{-\theta_1} & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ R_1 - Z_1 & (Z_{C1} + Z_1)/e^{-\theta_1} & -Z_{C1}/e^{-\theta_1} & 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & -Z_{C1} & Z_{L1} + Z_{C2} + Z_{C1} & -Z_{C2} & 0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & -Z_{C2} & Z_{L2} + Z_{C3} + Z_{C2} & -Z_{C3} & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0 & -Z_{C3} & Z_{L3} + Z_{C4} + Z_{C3} & -Z_{C4} & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 & -Z_{C4} & Z_{L4} + Z_{C5} + Z_{C4} & -Z_{C5} & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 & -Z_{C5} & Z_{L5} + Z_{C6} + Z_{C5} & -Z_{C6} & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 & Z_{C6}e^{-\theta_2} & (Z_2 - Z_{C6})e^{-\theta_2} & Z_2 + R_2 \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & Z_{C6}/e^{-\theta_2} & (-Z_{C6} - Z_2)/e^{-\theta_2} & R_2 - Z_2 \end{bmatrix} \quad (4)$$

3 Equivalent model of the constituting elementary structures

Step 1 splitted the structure into 2 parts constituted by interconnects and vias. In this case, we start to analyse the interconnects which are two microstrips located on the top and bottom layer of the structure. The equivalent graph topology and the computational equations are developed by applying Branin's method. Then the equivalent model of via through top layer to bottom layer is established and computed with Kron's tensorial network analysis.

3.1 Interconnect line modelling

The graph topology of the interconnects interacted with lands is depicted by applying Branin's method in Fig. 4, C_1 and C_6 are the lands located on the top and bottom layer respectively connect to interconnect1 and interconnect2. Associating with the first Kirchoff's law, the following relations can be developed from the graph topology of the interconnects.

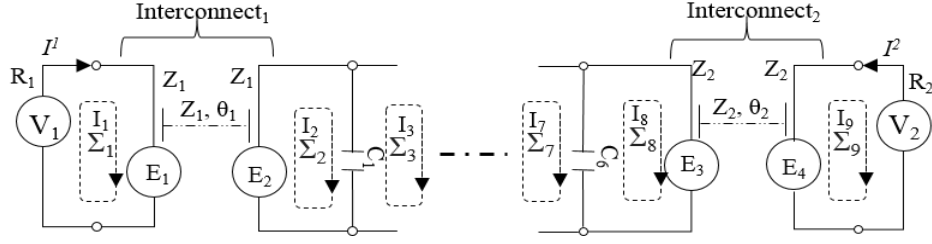


Figure 4: Equivalent Branin model topology for interconnects.

$$\begin{cases} \sum 1: V_1 = (R_1 + Z_1) I_1 + E_1 \\ \sum 2: -Z_1 I_2 + E_2 = Z_{C1} (I_2 - I_3) \end{cases} \quad \begin{cases} \sum 8: Z_2 I_8 + E_3 = Z_{C6} (I_7 - I_8) \\ \sum 9: V_2 = (R_2 + Z_2) I_9 + E_4 \end{cases} \quad (5)$$

The induced Branin's voltage sources (E_k) elaborated in Fig. 4 are expressed as:

$$\begin{cases} E_1 = [Z_{C1} (I_2 - I_3) - Z_1 I_2] e^{-\theta_1} \\ E_2 = [V_1 + (Z_1 - R_1) I_1] e^{-\theta_1} \\ E_3 = [V_2 + (Z_2 - R_2) I_2] e^{-\theta_2} \\ E_4 = [Z_{C6} (I_7 - I_9) + Z_2 I_8] e^{-\theta_2} \end{cases} \quad (6)$$

Substituting equation (6) into (5), it becomes the form as (7) and (8) without any Branin's voltage sources (E_k). If we want to analyse interconnects without lands connected, C_1 and C_6 will be shorted, and the equation (8) will be used. However, in this case, Equation (7-8) will be applied to connect to equations (11) which is the via part.

$$\sum 1,2 \begin{cases} V_1 \\ V_1 \end{cases} = \begin{bmatrix} Z_1 + R_1 & (Z_{C1} - Z_1) e^{-\theta_1} & -Z_{C1} e^{-\theta_1} \\ R_1 - Z_1 & (Z_{C1} + Z_1) / e^{-\theta_1} & -Z_{C1} / e^{-\theta_1} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \quad (7)$$

$$\sum 8,9 \begin{cases} V_2 \\ V_2 \end{cases} = \begin{bmatrix} Z_{C6} e^{-\theta_2} & (Z_2 - Z_{C6}) e^{-\theta_2} & Z_2 + R_2 \\ Z_{C6} / e^{-\theta_2} & (-Z_{C6} - Z_2) / e^{-\theta_2} & R_2 - Z_2 \end{bmatrix} \begin{bmatrix} I_7 \\ I_8 \\ I_9 \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} V_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} Z_1 e^{-\theta_1} & Z_1 + R_1 \\ -Z_1 / e^{-\theta_1} & R_1 - Z_1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \begin{bmatrix} V_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_2 e^{-\theta_2} & Z_2 + R_2 \\ -Z_2 / e^{-\theta_2} & R_2 - Z_2 \end{bmatrix} \begin{bmatrix} I_8 \\ I_9 \end{bmatrix} \quad (9)$$

3.2 Via elements modelling

The graph topology of the via system is depicted in Fig. 5, C_1 , C_6 , $C_{2..5}$, $L_{1..5}$ are the top layer land capacitance, bottom layer land capacitance, anti-pads capacitance and via hole inductance respectively in this multilayer structure. The Kron's method associating the first Kirchoff's law is employed in this via system structure.

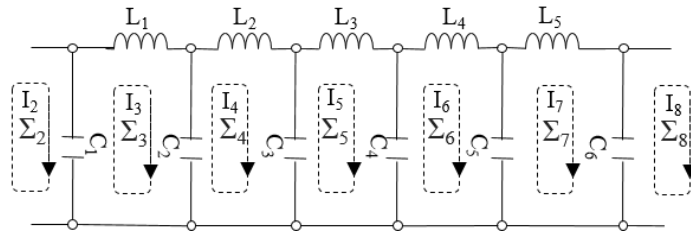


Figure 5: Equivalent topology of Kron's model for via system.

$$\begin{cases} \sum 3: I_3 Z_{L1} + (I_3 - I_4) Z_{C2} - (I_2 - I_3) Z_{C1} = 0 \\ \sum 4: I_4 Z_{L2} + (I_4 - I_5) Z_{C3} - (I_3 - I_4) Z_{C2} = 0 \\ \sum 5: I_5 Z_{L3} + (I_5 - I_6) Z_{C4} - (I_4 - I_5) Z_{C3} = 0 \\ \sum 6: I_6 Z_{L4} + (I_6 - I_7) Z_{C5} - (I_5 - I_6) Z_{C4} = 0 \\ \sum 7: I_7 Z_{L5} + (I_7 - I_8) Z_{C6} - (I_6 - I_7) Z_{C5} = 0 \end{cases} \quad (10)$$

$$[Z] = \begin{bmatrix} -Z_{C1} & Z_{L1} + Z_{C2} + Z_{C1} & -Z_{C2} & 0 & . & . & 0 \\ 0 & -Z_{C2} & Z_{L2} + Z_{C3} + Z_{C2} & -Z_{C3} & 0 & . & . \\ . & 0 & -Z_{C3} & Z_{L3} + Z_{C4} + Z_{C3} & -Z_{C4} & 0 & . \\ . & . & 0 & -Z_{C4} & Z_{L4} + Z_{C5} + Z_{C4} & -Z_{C5} & 0 \\ 0 & . & . & 0 & -Z_{C5} & Z_{L5} + Z_{C6} + Z_{C5} & -Z_{C6} \end{bmatrix} \quad (11)$$

Step 2 makes the combination with the matrix expression of (7-8) and (11), the impedance matrix expressed in (4) presented in section 2.2 can be derived. Therefore, the voltage and current vectors can be linked by equation (12).

$$\begin{bmatrix} V_1 \\ V_1 \\ 0 \\ \vdots \\ 0 \\ V_2 \\ V_2 \end{bmatrix} = [Z] \times \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_7 \\ I_8 \\ I_9 \end{bmatrix} \quad (12)$$

The form can be rewritten as (13):

$$[I(j\omega)] = [Z(j\omega)]^{-1} \times [V(j\omega)] \quad (13)$$

This equation is solved by Python programming, the current results can be applied to the S parameter and input impedance extraction with simplified equations (14-16) from K. Kurokawa [13]:

$$S_{11} = \frac{b_1}{a_1} \big|_{a_1=0} = 1 - \frac{2R_1 I^1}{V_1} \quad (14)$$

$$S_{21} = \frac{b_2}{a_1} \big|_{a_2=0} = -\frac{2R_2 I^2}{V_1} \quad (15)$$

$$Z_{in} = \frac{V_1}{I_1} - R_1 \quad (16)$$

In (14) and (15), a_k and b_k ($k = \{1,2\}$) are the wave powers, and I^1 and I^2 indicates the branch currents at the mesh 1 and 9.

4 KB validation and simulation results

To materialize the KB method efficiency, the POC was designed and simulated with the commercial tool ADS[®] from Keysight Technologies[®]. The KB model computations were performed with Python[®] programming by taking via parameters in Table 1 from ADS extraction. Mean values of these parameters, extracted with the frequency band from 20 GHz – 50 GHz in ADS, were used.

These value can also be computed with the closed-form equations from [5][14-15] which has higher accuracy than ADS extraction.

Top Pad (C_1)	Anti-pad through GND plane (C_2)	Anti-pad (C_3)	Anti-pad (C_4)	Via-hole1 (L_1)	Via-hole2 (L_2)
31 fF	53 fF	13.7 fF	14.2 fF	0.89 nH	0.89 nH
Via-hole3 (L_3)	Via-hole4 (L_4)	Via-hole6 (L_5)	Anti-pad through power plane (C_5)	Anti-pad (C_6)	
0.89 nH	0.89 nH	0.89 nH	45 fF	38.5 fF	

Table 1: Via model parameters.

The comparison results of the reflection, transmission and impedance parameters S_{11} , S_{21} , Z_{in} are displayed in Fig. 6 and Fig. 7. The excellent agreement with ADS simulations were obtained from 20 GHz to 50 GHz.

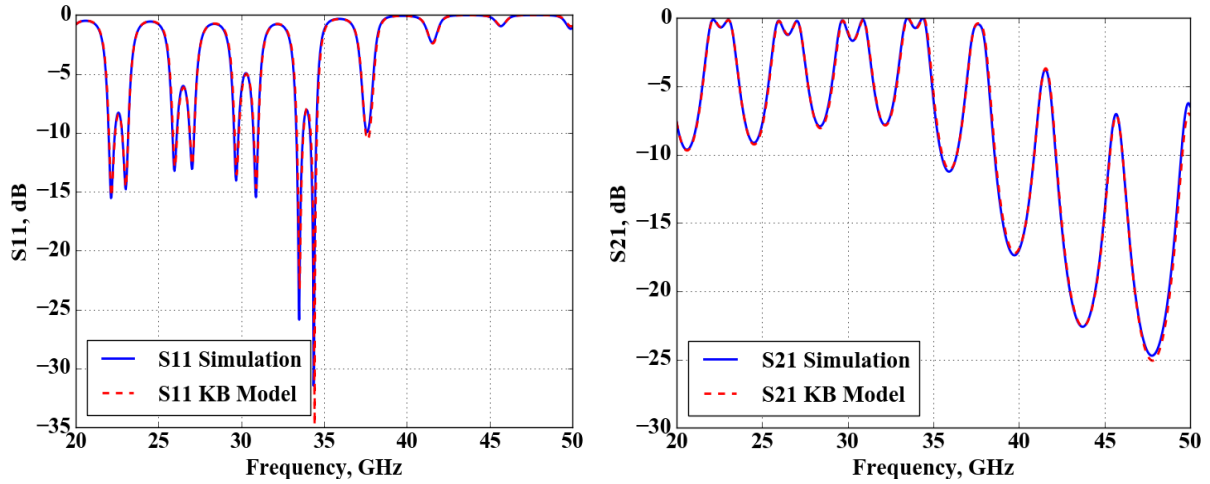


Figure 6: Comparison of KB model computation and ADS® simulation reflection and transmission coefficients.

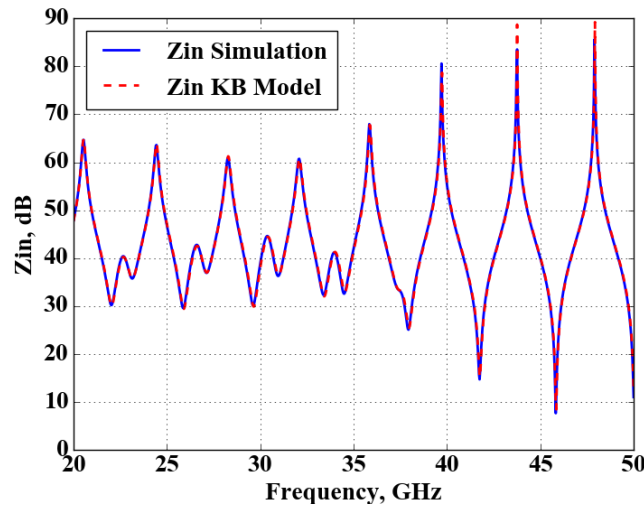


Figure 7: Input impedance comparison.

5 Conclusion

An unconventional KB-formalism modelling of 6-layer PCB [10] is established. The KB model feasibility is validated with S-parameters analysis simulated with ADS®. Compare to ADS, this model is mainly advantageous in term of the adaptability to complex structures, the computation speed and low cost.

First and foremost, the diakoptic analysis method is applied to separate the structure to interconnects and vias in order to get a deep understanding and processing. Secondly, the equivalent graph topologies are established based

on the structure. Finally, the impedance matrix and computation are developed by Python programming. Within this structure, the KB total calculation time is 0.3s. In this paper, the L , C values of via system are extracted from ADS. However, the accuracy mainly depends on the calculation of LC values for via cells [5][14-15] constituting in this model network. The next step of our work will focus on the accuracy, speed and meshing compare to 3D EM solvers.

Acknowledgements

This research work was supported by Euripides ²Eureka Program funded by the research project “Embedded Die Design Environment & Methodology for Automotive Applications (EDDEMA)”, 2015-2018.

References

- [1] <http://www.itrs2.net/itrs-schedule.html>
- [2] A. E. Ruehli and A. C. Cangellaris, “Progress in the methodologies for the electrical modeling of interconnects and electronic packages,” *Proceedings of the IEEE*, Vol. 89, No. 5, pp. 740-771, 2001.
- [3] B. J. LaMeres, “Characterization of a Printed Circuit Board via,” PhD thesis, University of Colorado, 2000.
- [4] E. Laermans, J. De Geest, D. De Zutter, F. Olyslager, S. Sercu and D. Morlion, “Modeling complex via hole structures,” *IEEE Trans. Advanced Packaging*, Vol. 25, No. 2, pp. 206-214, May 2002.
- [5] S. H. Hall, G. W. Hall, J. A. McCall, “High-speed digital system design: a handbook of interconnect theory and design practices,” New York: Wiley, 2000.
- [6] F. H. Branin, “Transient analysis of lossless transmission lines,” *Int. J. Proceedings of the IEEE*, Vol. 55, No.11, pp. 2012-2013, Nov. 1967.
- [7] O. Maurice, A. Reineix, P. Durand and F. Dubois, “Kron’s method and complexes for magnetomotive and electromotive forces,” *Int. J. Applied Mathematics*, Vol. 44, No. 4, pp. 183-191, 2014.
- [8] B. Ravelo, O. Maurice and S. Lalléchère, “Asymmetrical 1:2 Y-tree interconnects modelling with Kron-Branin formalism,” *Electronics Letters*, Vol. 52, No. 14, pp. 1215-1216, July 2016.
- [9] B. Ravelo and O. Maurice, “Kron-Branin modelling of Y-Y-tree interconnects for the PCB signal integrity analysis,” *IEEE Trans. Electromagnetic Compatibility*, Vol. PP, No. 99, pp. 1-9, 2016.
- [10] T. Nikola, “Standard 4-and 6-Layer PCB stackups,” White Paper, Data Response, Available Online, 2013, <http://www.bitweenie.com/listings/standard-4-and-6-layer-pcb-stackups/>.
- [11] H.H. Happ, “Diakoptics and Networks,” Academic Press, 1971.
- [12] I. J. Bahl and D. K. Trivedi, “Designers guide to microstrip line,” *Int. J. Microwaves*, Vol. 16, No. 5, pp. 174-182, 1977.
- [13] K. Kurokawa, “Power waves and the scattering matrix,” *Int. J. IEEE transactions on microwave theory and techniques*, Vol. 13, No. 2, pp. 194-202, Mar. 1965.
- [14] Y. Shang, C. Li and H. Xiong, “One method for via equivalent circuit extraction based on structural segmentation,” *Int. J. Computer Science Issues*, Vol. 10, No. 3, pp. 18-22, May 2013.
- [15] M. E. Goldfarb and R. A. Pucel, “Modeling via hole grounds in microstrip,” *Int. J. IEEE Microwave and Guided Wave Letters*, Vol. 1, No. 6, pp. 135-137, 1991.