ÉNERGIE ET RADIOSCIENCES

Energy saving in Analog to Digital Convertors: how Multi-Coset Non Uniform sampling scheme can help

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Introduction

The ever-growing increase of frequency bandwidths of telecommunications systems have been putting a huge constraint on Analog to Digital Convertors (ADC). This constraint originates from the Shannon-Nyquist sampling law: to prevent any overlap sampling have to be performed at the Shannon-Nyquist rate which has to equal at least twice the transmitted bandwidth. As a result, the larger the bandwidth, the higher the sampling frequency and the higher the energy consumption of the ADCs. To cope with this issue, especially in non-contiguous bandwidths (ie no full bands containing holes [2] – one would refer to sparse signal spectrums), one solution is to use non uniform sampling schemes (NUSS) what makes possible the update of the sampling frequency according to the spectral occupancy rate. Given the Multi-Coset (MC) NUSS this paper proposes an original criteria (namely AliasMin mode) to lower the spectrum side lobes of signals when using NUSS. Following this idea, it is shown that the sampling frequency of ADCs can be updated according to the spectrum occupancy rate (ie sparsity rate) to mitigate the energy consumption of ADCs.

1. Non Uniform sampling and Multi-Coset scheme

Non Uniform Sampling Schemes (NUSS) have been proposed for a long time due to their nice property to lower the replica of signal spectrums [1]. Several NUSS exist and can be classified in two categories: deterministic schemes and random schemes. The first category gathers schemes whose sampling times are perfectly known (not random) what is not true in the second one. Considering that the Multi-Coset belongs to the first category and that Jittered Random Sampling (JRS) and Additive Random Sampling (ARS) belong to the second one, the schedule of sampling strategies is sketched on Figure 1.

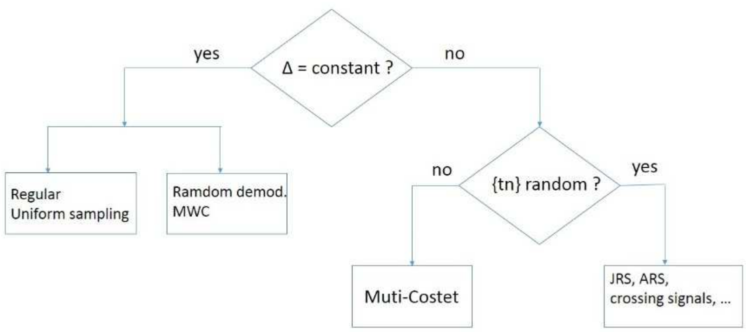

Figure 1 : Sampling schemes classification (Δ: sampling time, $\{t_n\}$ set of samples times)

Multi-Coset (MC) scheme is an attractive NUSS because it leads to a sampling frequency lower to Shannon-Nyquist one and has a good reconstruction quality if the associated pattern (see below) is well chosen. Figure 2 illustrates the principle of MC compared to uniform sampling scheme (Nyquist samples). MC is a periodic non uniform sampling scheme whose sampling pattern is the same all along the signal itself: the process selects *p* samples among *L*. The *p* samples (7 on Figure 2) are chosen according to a periodic pattern (0, 2, 5, etc.).

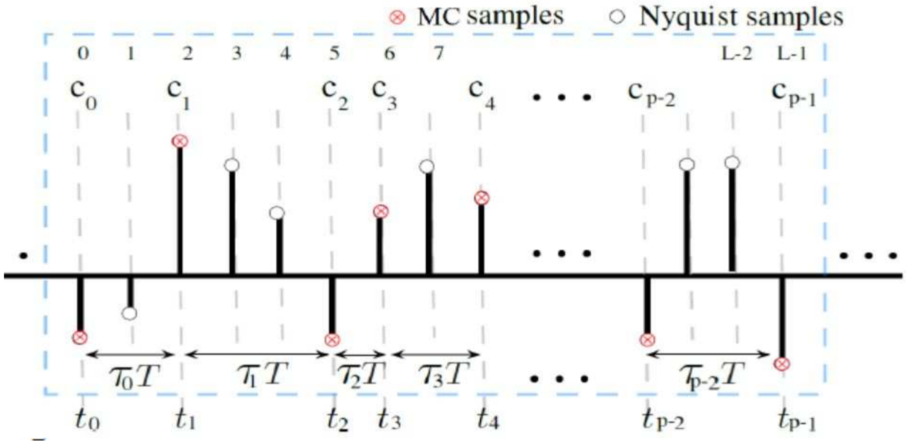

Figure 2 : Multi-Coset principle

As said, the choice of the pattern (and its samples) is a key parameter in the MC-NUSS as the resulting spectral regrowth impact by large the reconstruction quality, the gain in the sampling frequency and as a consequence the energy gain of the ADC.

The resulting pattern which gathers all sampling times after MC sampling is given as follows:

$$u_{mc}(t) = \sum_{k=0}^{\infty} \alpha_k \delta(t - kT),$$

where

$$\alpha_k = \alpha_{(k+iL)}, 0 \leqslant k \leqslant p-1, 0 \leqslant i < \infty,$$

and T is the sampling period. This signal can be written as a Fourier series:

$$u_{mc}(t) = \sum_{n=\mathbb{Z}} A_n \exp(j2\pi \frac{n}{LT} t)$$

where

$$\begin{aligned} A_n &= \frac{1}{LT} \int_0^{LT} u_{mc}(t) \exp(-j2\pi \frac{n}{LT} t) dt \\ &= \frac{1}{LT} \sum_{k=0}^{L-1} \alpha_k \exp(-j2\pi \frac{n}{L} k). \end{aligned}$$

The Fourier Transform of $u_{mc}(t)$ is then given by :

$$U_{mc}(f) = \sum_{n=\mathbb{Z}} A_n \delta(f - \frac{n}{LT}).$$

As a result, the spectrum of any signal *x(t)* sampled with MC scheme is given by :

$$X_{mc}(f) = \sum_{n=-\infty}^{+\infty} A_n X(f - \frac{n}{LT}),$$

where X(.) is the spectrum of x(t). Then, it is easy to show that the spectrum of $X_{mc}(f)$ depends on A_n which depends on α_k , L and p. That is to say that a good choice of *p* and *L* results in good spectral properties of MC sampling scheme.

The MC scheme can be divided into three steps (i) sampling at frequency *1/T* (ii) slicing the sampling times into pieces of *L* samples (iii) keeping *p* samples per slice. Considering that the signal *x(t)* is of length γLT (and truncated with a rectangular shape function), the final signal is given by :

$$w_{mc}(t) = w_{rect}(t) u_{mc}(t).$$

Then, in the frequency domain,

$$W_{mc}(f) = \sum_{n=-\infty}^{+\infty} A_n sinc\left(\pi\gamma LT \left(f - \frac{n}{LT}\right)\right)$$

where A_n have been defined previously. Note that the rectangular shape can be changed to Hamming, Hanning or Blackman.

We first consider the *Burst* mode where the p samples are the first ones of the window of size L. Figure 3 illustrates the shape of the spectrum for L=32 and L=22 and for different values of γ. It is seen that the choice of the samples influence by large the spectrum quality and regrowth. Same result can be obtained according to the *Rand* mode for which the samples are chosen randomly in the window of size L.

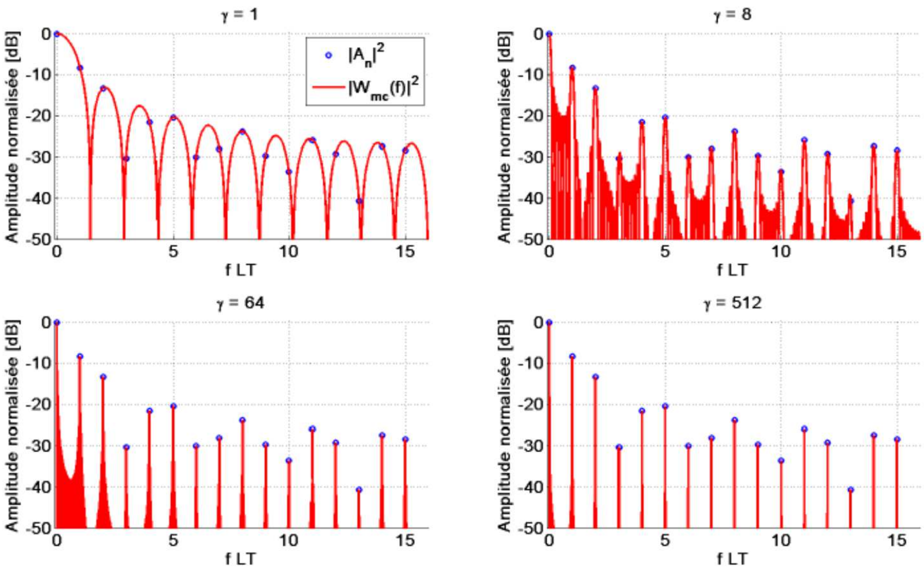

Figure 3 : $W_{mc}(f)$ for L=32 and p=22 (Burst Mode)

As a result, we proposed the Alias Min algorithm which selects the *p* sampled which minimize A_n. This algorithm is called "AliasMin".

2. AliasMin algorithm and simulation results

The Alias Min algorithm (Figure 5) selects p samples among L similarly to SFS (Sequential Forward Selection) in [4]. Figure 4 illustrates Alias Min performance. The more γ increases, the more frequencies are well localized (at multiples of *1/LT*).

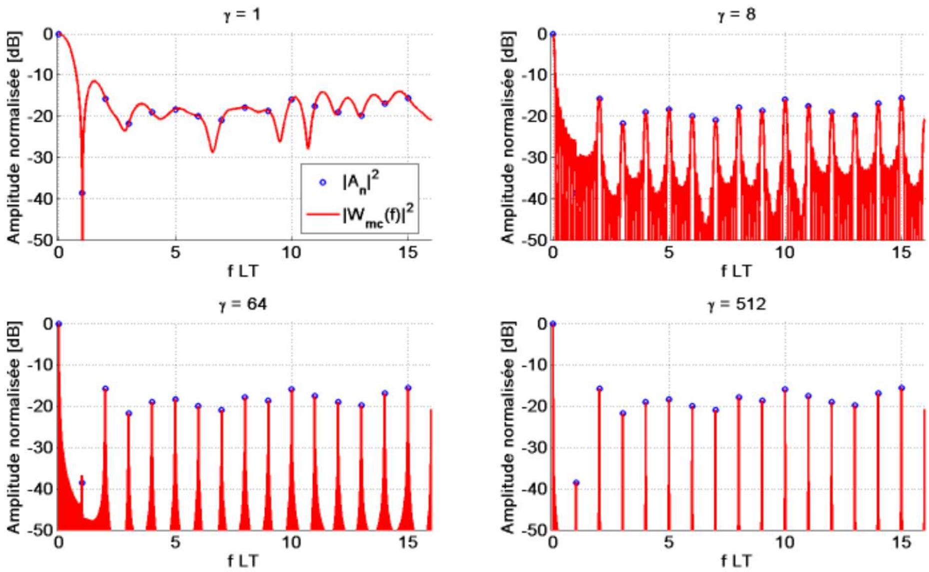

Figure 4 : Alias min results for L=32 and L=22

Algorithme 1 : Algorithme *AliasMin*

1 **Entrées :** L , p *et* $\mathcal{K}$

Sortie : $\mathcal{C}$

Initialisation :

$\mathcal{C} \leftarrow \varnothing$

$\mathcal{C}_s \leftarrow \{0, 1, ..., L-1\}$

$m \leftarrow 1$

$F \leftarrow$ une matrice de taille $(\frac{L}{2} \times L)$ dont le $(l,q)^{\text{ème}}$ élément est $[\exp(-j2\pi lq/L)]_{l,q}$ pour $1 \leqslant l \leqslant \frac{L}{2}$ et $0 \leqslant q \leqslant L-1$

Traitement :

Tant que : $m \leqslant p$ **faire**

2 $n \leftarrow 1$

Tant que : $n \leqslant L-m+1$ **faire**

3 $v \leftarrow$ un vecteur de taille L ayant 1 aux positions correspondant à l'ensemble $\{\mathcal{C} \cup \mathcal{C}_s(n)\}$ et zéro ailleurs.

$\mathcal{C}_{opt} \leftarrow \arg\min\,[\max(Fv)]$

$n \leftarrow n+1$

4 **Fin**

5 $\mathcal{C} \leftarrow \mathcal{C} \cup \mathcal{C}_{opt}$

$\mathcal{C}_s \leftarrow \mathcal{C}_s - \{\mathcal{C}_{opt}\}$

$m \leftarrow m+1$

6 **Fin**

Figure 5 : Alias Min algorithm

The "AliasMin" algorithm aims to mitigate the ratio $\Delta H = \frac{max\{|A_n|^2\}}{[|A_0|^2]}$. Figure 6 illustrates the algorithm for L=32 and different values of p.

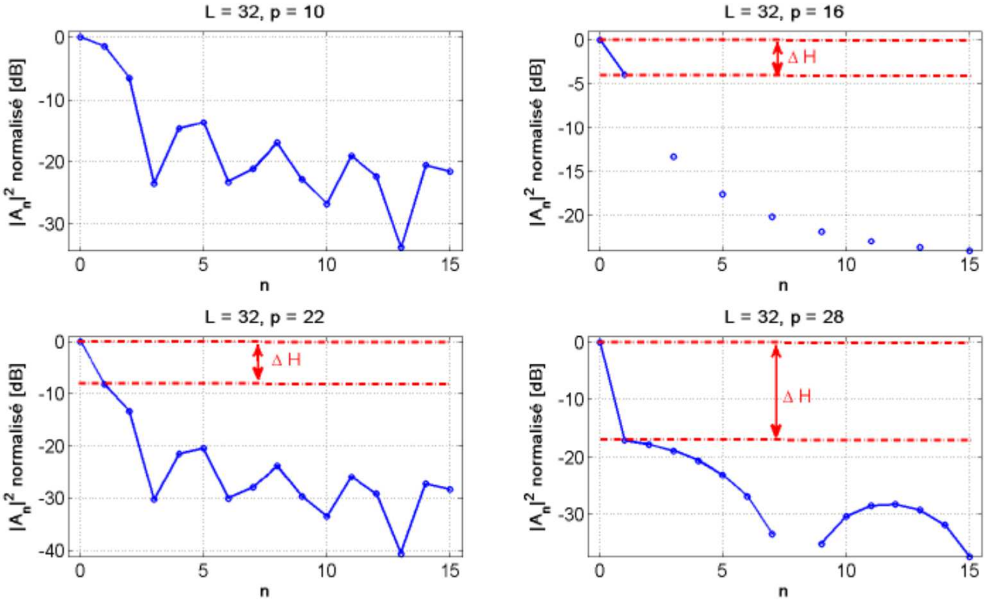

Figure 6 : Alias Min for different p values (L=32)

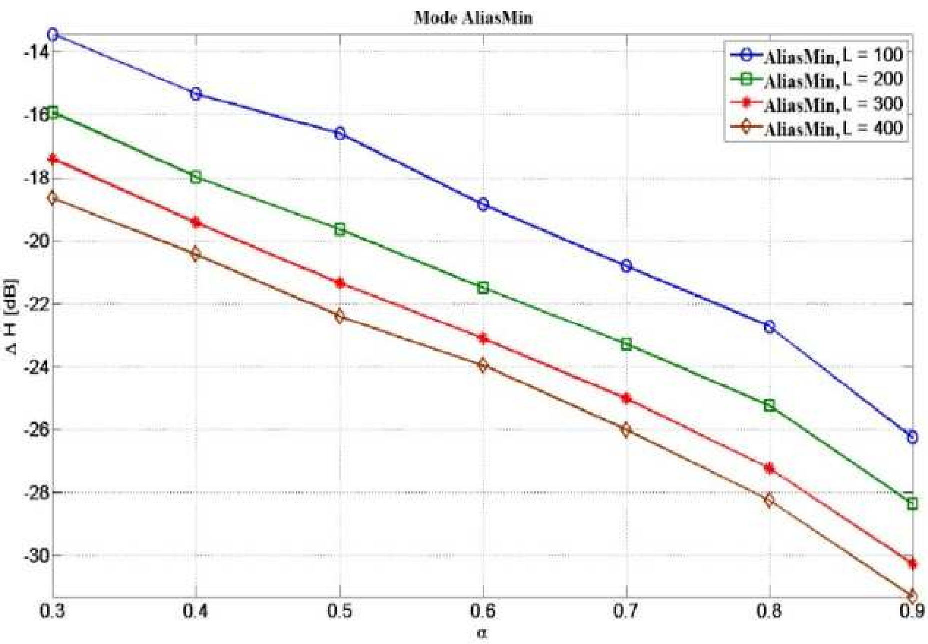

Figure 7 : Alias Min for different values of p and L (α=p/L)

3. SENURI sampler

Using Alias Min algorithm in the SENURI sampler which tunes the sampling frequency according to the sparcity of the signals [3], it has been shown that there is a benefit in using a cognitive engine. Figure 9 illustrates this gain compared to the case where the sampling frequency is always the same (ie equal to the Shannon frequency). SERUNI is based on a MC scheme and the quality of the spectrum sensing step depends on the spectrum quality driven by the Alias Min algorithm. Figure 8 sketches the main steps of the proposed non uniform sampler: position 1 refers to the adaptation step (tuning the sampling parameter according to the signal sparsity) and position 2 refers to the reconstruction step.

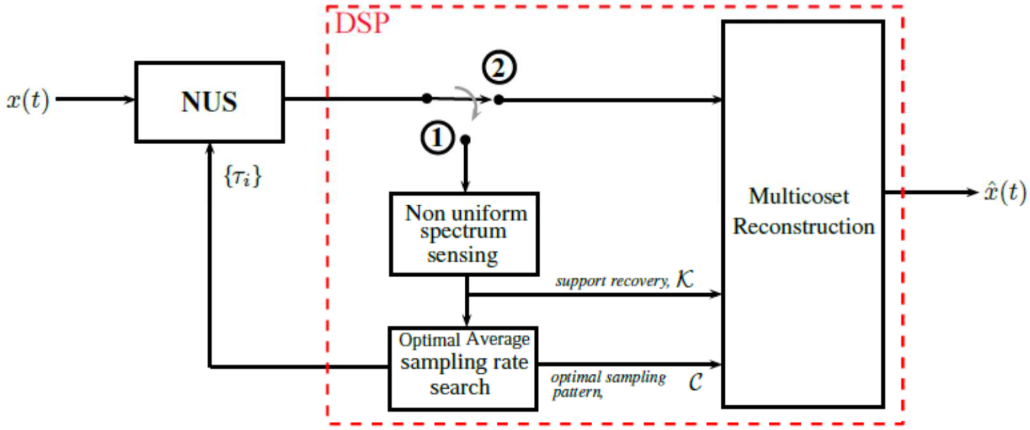

Figure 8 : Dynamic Non uniform sampler

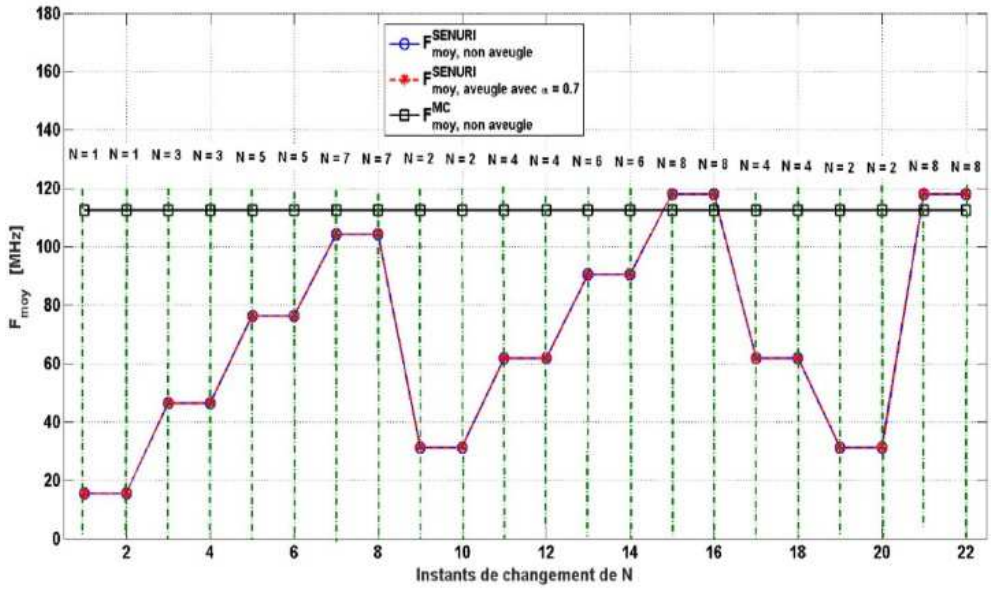

Figure 9 : sample frequency gain

4. Conclusions

In this paper, we showed that the choice of samples in non uniform schemes influence by large the quality of the spectral quality. When considering a cognitive scheme where the sampling frequency is tuned according to the sparsity of the frequency, the sampling frequency can be mitigated what save energy in the ADC.

5. References

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