ARTEMIS-P: un code de tracé de rayons en milieu anisotrope et applications en radio astronomie.

ARTEMIS-P: A general Ray Tracing code in anisotropic plasma for radioastronomical applications.

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Abstract

The ARTEMIS-P code is a ray tracing code in magnetoionic plasma based on the Haselgrove equations [7], which computes the polarization state of the wave along the ray path. With the general form of the equations as well as their implementation scheme, it is able to perform ray tracing studies in any given magneto-ionic medium, such as planetary magnetospheres. The ARTEMIS-P code has already been used to study the propagation 'over the horizon' of Saturn Electrostatic Discharges through the Kronian magnetosphere. We present here the principle of the ARTEMIS-P code and these two applications.

Keywords / Mots-clés
Ray tracing; Propagation in anisotropic medium; Magnetoionic theory
Tracé de rayons; Propagation en milieu anisotrope ; Thorie magnéto-ionique

1 Introduction

Ray tracing techniques are important tools in the study of radio waves propagation. Based on the calculation of the trajectory of electromagnetic energy through a given medium (ionosphere, solar wind, tokamak . . .), these techniques provide approximate solutions of Maxwell's equations in this medium. Since the 50's, many different ray tracing techniques were developed, from geometrical ones, as Poeverlein method [9], to numerical ones, as Haselgrove method [6], [7]. This latter is probably the most useful technique for a general study of the propagation of radio waves in the terrestrial ionosphere. The general form of the Haselgrove equations [7] allows us to extend their application not only in the terrestrial ionosphere but also in any anisotropic plasma which is a common medium in the solar system, from the Sun to planetary magnetospheres. Planetary magnetospheres result from the interaction between the solar wind and the planetary magnetic field, which dominates the dynamics of charged particles. They are complex structures where interplanetary plasma and magnetic field are in interaction. Planetary radio emissions are produced by the precipitation of plasma electrons along the magnetic field lines, close to the magnetic poles. After their emission, the radio waves propagate through this inhomogeneous and anisotropic medium. In order to compute the refractive effects sustained by such radio waves through planetary magnetospheres, we are developing ARTEMIS-P (Anisotropic Ray Tracer for Electromagnetism in Magnetosphere, Ionosphere and Solar wind including Polarization), a general three-dimensional ray tracing code based on the Haselgrove equations, which also include computation of the polarization state along the ray path. The scope of this paper is to briefly describe the numerical scheme of ARTEMIS-P code and given some examples of application in the Saturn ionosphere and magnetosphere.

2 Description of ARTEMIS-P code

2.1 Ray tracing based on the Haselgrove equations

This code is based on the Haselgrove equations described in [7], which are differential equations in position **r** and wave vector **k** along the ray path. In anisotropic medium, the main difficulty arises from the non-collinearity of the wave vector and the ray path (Poynting vector **P**). In such a medium the refractive index n at frequency f is given by the magnetoionic theory [1] :

$$n^2 = 1 - \frac{2X(1-X)}{2(1-X) - Y^2\sin^2\theta \pm \sqrt{Y^4\sin^4\theta + 4Y^2(1-X)^2\cos^2\theta}} \tag{1}$$

Where $X = (f_p/f)^2$ with f_p the plasma frequency, $Y = f_c/f$ with f_c the cyclotron frequency and θ the angle between the wave vector **k** and the ambient magnetic field **B**. The sign $(-)$ corresponds to the eXtraordinary mode and $(+)$ to the Ordinary mode.

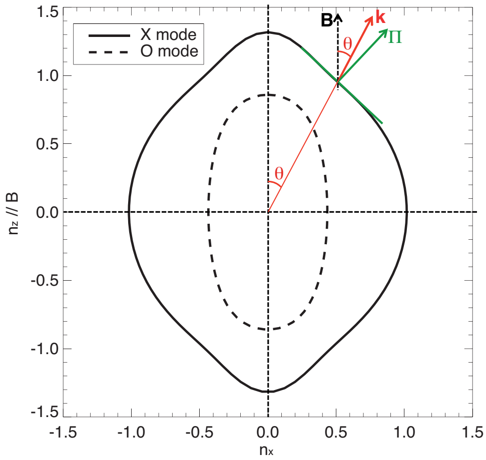

Figure 1: Definition of the wave vector **k** and Poynting vector **P** for the refractive index surface of X mode and O mode with $X = 0.81$ and $Y^2 = 4.41$.

If we consider the refractive index surface described by a vector of length n and direction **k**, the Poynting vector **P** is defined by the normal of the surface at the intersection with the vector **k**, as illustrated in figure 1.

In [6], the author derives a set of Hamiltonian equations in **r** and **k** along the ray path from the Fermat's principle and the property of reciprocal surfaces of the refractive index surface and the wave front.

Equations implemented in ARTEMIS-P (coming from[7]) are more suitable for numerical integration and propagation in isotropic medium. These differential equations are solved by a 4th order Runge-Kutta algorithm, with an integration step which is adjusted with respect to the gradient of refractive index along the ray. Equations of propagation are implemented in a general form and ARTEMIS-P only needs a description of the propagation medium and the initial characteristics of the ray supplied by the user.

2.2 Wave polarization along the ray path

The polarization of radio waves is defined by the direction of its electric field **E**. To characterize the propagating electromagnetic wave, the relative position of the electric field vector of the wave **E** with respect to the wave vector **k** has to be calculated. In a frame where the vector **k** is fixed, it amounts to find the components of the electric field **E**, which are extracted from Maxwell's equations in a given medium :

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + \frac{\omega^2}{c^2}\overline{\overline{\kappa}}\mathbf{E} = \mathbf{0} \tag{2}$$

where $\overline{\overline{\kappa}}$ is the dielectric tensor. In a cold magnetized plasma $\overline{\overline{\kappa}}$ is given by (see [5]):

$$\overline{\overline{\kappa}} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}$$

where

$$S = 1 - \frac{X}{1-Y^2}, \quad D = \frac{YX^2}{1-Y^2}, \quad P = 1 - X \tag{3}$$

In a frame where the local magnetic field **B** is along the $z-$axis, the wave vector **k** begin in the (x,z) plane and making an angle θ with **B**, the components of the electric field solutions of equation (2) are linked by (see [10]):

$$\frac{E_x}{(n^2-S)(n^2\sin^2\theta - P)} = \frac{E_y}{iD(n^2\sin^2\theta - P)} = \frac{E_z}{(n^2-S)n^2\sin\theta\cos\theta} \tag{4}$$

Thus the polarization of the wave only depends on the characteristics of the propagation medium and θ. In addition to the propagation computation, the ARTEMIS-P code can calculate the polarization ratios E_y/E_x, E_z/E_x and E_z/E_y in each point along the ray path. In radio astronomy, the polarization of a wave is often described with the Stokes parameters S, Q, U and V, where S is the flux density of the wave, Q and U characterize the linear polarization and V is the circular polarization degree. It can be shown that the Stokes parameter V can be related to the axial ratio $\rho = E_y/E_x$ when the longitudinal component of the polarization can be neglected by (see [8]):

$$V = \frac{2\rho}{1+\rho^2} \tag{5}$$

2.3 Mode coupling and limiting polarization

The magnetoionic theory predicts the existence of different mode of propagation of radio waves (see equation (1)), often called eXtraordinary mode and Ordinary mode, both characterized by a refractive index (n_O and n_X) and a polarization state (ρ_O and ρ_X). In an homogeneous medium, each mode propagates independently and its features are fixed by the medium. When the medium is slowly varying, Booker [2] has shown that in many cases this theory is still valid, and the features of the wave are varying along the ray path. However he has also shown that there exist regions in the ionosphere where the magnetoionic modes are not propagating independently. In these regions the propagation modes are 'coupled'. Furthermore Booker has shown in [2] that when a radio wave arrives in such a region its polarization is frozen and does not change across the coupling region. It is called the 'limiting polarization' effect.

Regions where mode coupling occurs are characterized in [3] by a difference between the refractive indices of the two modes smaller than there variations along the ray path:

$$|n_O - n_X| \leq \frac{1}{k}\left|\frac{\mathrm{d}n_O}{\mathrm{d}s}\right| \text{ or } \frac{1}{k}\left|\frac{\mathrm{d}n_X}{\mathrm{d}s}\right| \tag{6}$$

where $\mathrm{d}s$ is the path length and $k = 2\pi c/f$.

The ARTEMIS-P code computes the refractive indices of both modes and there derivatives along the ray path. As long as the criterion of the equation (6) is not satisfied, the polarization ratios are computed in each point of the ray and as soon as this criterion is satisfied, the polarization ratios are retained as long as the propagating modes are 'coupled'.

3 Propagation over the horizon of Saturn Electrostatic Discharges

Saturn Electrostatic Discharges (SED) are radio signatures of lightning flashes originating from Saturn atmosphere. Before the equinox in 2009, observations by Cassini/ RPWS (Radio and Plasma Waves Science) and Cassini/ISS (Imaging Science Subsystem) have shown strong correlation between the periodicity and occurrence of radio bursts and cloud features localized at a planetocentric latitude of 35° South and an altitude of 2000 km below the altitude of the peak of electron density of the ionosphere. When Cassini was located in the morning sector, the cloud system appeared on the nightside and disappeared on the dayside, but the detection of SED started before the cloud system could actually be seen by the Cassini camera (figure 2a). This effect is called the 'over the horizon' effect, as the radio horizon extends beyond the visible horizon. In [11], this effect has been qualitatively attributed to propagation effects through the nightside ionosphere, where electron density varies with the local time. Indeed, if a radio wave is emitted at a frequency lower than the local plasma frequency, it can then be trapped under the ionosphere and ultimately escape where the local plasma frequency becomes lower than the wave frequency and then it can reach the Cassini spacecraft.

We have used the ARTEMIS-P code to compute the path of propagation of radio waves through a realistic model of Saturn ionosphere. Over the horizon effect is detected by the RPWS instrument in a frequency range of 2 MHz to 16 MHz, which is high above the typically local cyclotron frequency ($f_c \sim 500$ kHz). We can neglect in this frequency range the effect of the kronian magnetic field on the propagation of radio waves. The variation of electron density with the local time is obtained by the study of the low frequency cut-off of the SED, studied in [4]. ARTEMIS-P propagates radio rays in the frequency range [2-16] MHz at several local times of the source during one storm episode. We then simulate an observed dynamic spectrum as illustrated in figure 2b. The grey levels in the simulation correspond to the mean number of reflections under the ionosphere before the escape of the rays that ultimately reach the spacecraft. For some geometric configurations, we can observe an extent before the horizon, which corresponds to a higher rate of reflection than in the center of the episode. This study shows that the hypothesis of trapped radio rays under the ionosphere is consistent and also shows the key-role of the peak electron density profile versus local time. A statistical study of the occurrences of the phenomenon on Cassini data and simulations of this occurrence are in progress.

4 Conclusion

The ARTEMIS-P code is a ray tracing code in magnetoionic plasma based on the Haselgrove equations, which computes the polarization state of the wave along the ray path and taken into account the limiting polarization in regions of mode coupling. The studies of the propagation 'over the horizon' of the SED and the refraction of the SKR through the Saturn magnetosphere shows the interest and the flexibilty of this code. The ultimate aim of the developement of the ARTEMIS-P code is to create a general, flexible, fast and user-friendly ray tracing tool which could be apply on a large panel of astrophysical studies (Quasi-Periodic bursts in jovian magnetosphere, planetary lightnings, propagation in auroral cavities, attenuation lanes, ...)

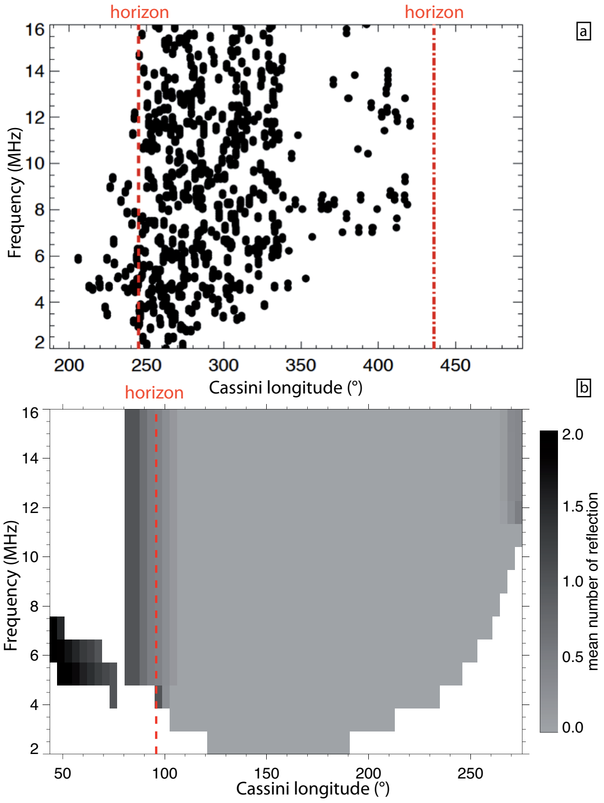

Figure 2: a) Dynamic spectrum of the SED episode during the day 04/10/2009. Black dots correspond to SED detections, red dashed lines to the Cassini spacecraft longitude when the cloud system cross the visible horizons. b) Simulation of a dynamic spectrum of a SED episode. Grey levels correspond to the mean number of reflections sustained by the rays before reaching Cassini and red dashed line to the Cassini longitude when the cloud system crosses the visible horizon.

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