Sur une analyse préliminaire de la modélisation à petite échelle de panneaux composites : arrangement périodique de fibres cylindriques circulaires

On a preliminary analysis of the electromagnetic small-scale modeling of composite panels: periodic arrangement of circular cylindrical fibers

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Résumé

Nous nous intéressons au contrôle non-destructif électromagnétique de panneaux composites fibreux, en hypothèse petite échelle, la longueur d'onde de l'excitation étant proche de la dimension des structures (cylindres circulaires) en réseau 1-D à l'intérieur de chaque nappe des panneaux, le milieu effectif standard ne tenant plus. Les travaux, préliminaires ici, s'appuient sur des recherches princeps en poro-élasticité et photonique, mais ils entendent s'appliquer à des structures fibres de carbone ou de verre pour lesquelles certaines sondes peuvent apprécier les cylindres d'une nappe d'une manière quasi individuelle.

1 General scope of the investigation

The contribution is about the electromagnetic (em) modeling and imaging of damaged, or better said, disorganized periodic structures. That is, a certain pattern within an elementary subdivision (cell) is repeated in the other cells of the structure into certain directions of space. This repetition is affected (disorganized) by changes of material properties and/or geometry of the constitutive parts, in one cell or several. Collecting em responses of such disorganized structures should yield images that exhibit location of the damaged zone(s).

Application is Non-destructive Testing (NdT) of synthetic panels of fiber composites, as, e.g., in aeronautic and automotive parts. At a first level of modeling, they can be viewed as piles of planar plates one over the other, each made of a regular linear arrangement of long cylinders with same circular sections, all oriented into the same direction (fibers) and whose constitutive material differs from the embedding material (matrix) which they are reinforcing. Then, for conductive panels (e.g., carbon-based), imaging should be in the low-frequency regime of eddy currents, and for lossless or weakly lossy panels (e.g., glass-based), in the high-frequency regime of microwaves.

2 Large- and small-scale modeling

Two hypotheses can be made at modeling stage. At large scale (large enough local wavelength in propagative regimes or skin depth in diffusive ones vs. key geometric features), locally averaged tensor parameters characterize each plate and are inserted into Maxwell's PDE [1]. [*We do not consider this case here.*] At small scale (small enough local wavelength or skin depth vs. geometry), the assumption is that each cell is containing one unique circular cylinder with due repetition, the orientation of the cylinders changing from one plate to the next. Then, since a given (undamaged) plate can be seen as infinitely periodic if of large lateral extent, it behaves like an infinite array, prone to Floquet-related modeling, to be weighted in vs. the limited spatial extent of the probing fields generated by most NdT probes (coils, dipoles, etc.), which means a finite number of cylinders effectively interacting. [*We consider this case here.*]

3 The main theoretical analysis

The analysis borrows a lot from pioneering poro-acoustics and elasticity (infinite extent case) [2] and photonics (limited extent case) [3] analyses. Yet it is tailored to the peculiar em configuration of concern, notably when highly-conductive (but not impenetrable) carbon fibers are immersed in a dielectric polymer and probed in MHz range, the preliminary case of TE- and TM-polarized planar incident fields impinging upon one single slab being focused onto.

The structure under scrutiny is sketched in Figure 1. A set of cylinders parallel to each other and directed in the y direction is embedded in a planar plate infinite in both x and y directions with interfaces Γ_a $(z = a)$ and Γ_b $(z = b)$. The cylinders are arranged periodically in the x direction with period d. Each is of radius c. So, each cell, as shown in the figure, has a width of d and height of $L = a - b$, the reference (unit) cell being the one displayed at the center. Correspondingly, the space is divided into subspaces $R_0^{\pm}$, R_1 and R_2. Otherwise, all materials are linear isotropic, possibly lossy (save the upper half-space), with ϵ_j and μ_j, $j = 0, 1, 2$, as permittivities and permeabilities.

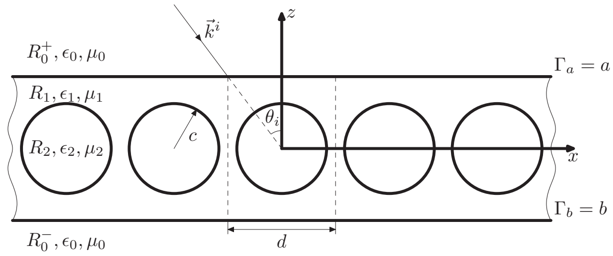

Figure 1: Plate including a periodic set of cylinders in x direction with plane of incidence as $x - z$ plane.

The TM-polarized incident plane wave with plane of incidence $x - z$ and obliquely impinging with θ^i angle upon the plate is such that its electric field reads as $\vec{E}_{inc} = \hat{y}E_{inc}e^{i[k_x^i x - k_z^i(z-a)]}$ with implied time-harmonic dependence $e^{i\omega t}$. $\vec{k^i}$ is the wave number vector of the incident wave, with absolute value k^i, and $k_x^i = k^i \sin\theta^i$, $k_z^i = k^i \cos\theta^i$, are its x- and z-components of $\vec{k^i}$. Wave numbers of regions $R_0^{\pm}$, R_1 and R_2 are k_0, k_1 and k_2 respectively, all of them satisfying the dispersion relation $k_j = \omega\sqrt{\epsilon_j\mu_j}$, $j = 0, 1, 2$. We will let whether necessary $\vec{k_j} = \alpha_j\hat{x} + \beta_j\hat{z}, j = 0, 1, 2$, α_j and β_j as x and z components of $\vec{k_j}$. Let us notice here that the analysis (TM- or E-polarization) is also suitable to TE- or H-polarization cases, via application of electromagnetic duality, the field of interest then being the magnetic one, and permittivity/permeability being exchanged (this does not mean that the numerics and the em behavior will be similar).

The particular feature of the problem is the transverse periodicity of the inclusions in region R_2. According to the Floquet theorem, this periodicity and the plane wave nature yield the well-known relation $E_{jy}(x+d, z) = E_{jy}(x, z)e^{i\alpha_0 nd}$, $j = 0, 1, 2$, where E_{jy} are the fields in Region $R_0^{\pm}$, R_1 and R_2, denoting fields in region R_0^+ and R_0^- as E_{0y}^+ and E_{0y}^- to separate them. As for the magnetic field, $H_{jx} = -\frac{1}{i\omega\mu}\frac{\partial}{\partial z}E_{jy}, j = 0, 1, 2.$

Using a plane wave expansion, we write the field in regions R_0^+ and R_0^- in the form

$$\begin{cases} E_{0y}^+(x,z) = \sum_{p\in\mathbb{Z}}(E_{inc}e^{-i\beta_{0p}(z-a)}\delta_{p0} + R_p e^{i\beta_{0p}(z-a)})e^{i\alpha_p x}, \\ E_{0y}^-(x,z) = \sum_{p\in\mathbb{Z}} T_p e^{i(\alpha_p x - \beta_{0p}(z-b))}; \end{cases} \tag{1}$$

R_p and T_p are the reflection and transmission coefficients of the plane wave indexed by p, δ_{p0} the Kronecker symbol, $\alpha_p = \alpha_0 + 2p\pi/d$, and $\beta_{jp} = \sqrt{k_j^2 - \alpha_p^2}, j = 0, 1$. Across the interfaces Γ_a and Γ_b, the tangential components of wave vector $\vec{k_j}$, $j = 0, 1$, are continuous, so we set α_p instead of α_{jp} in all equations.

In region R_1, according to superposition principle, the total field equals to the sum of the field scattered by the inclusions and the diffracted field in the plate. We write it in Cartesian coordinates as

$$E_{1y}^{\pm}(x,z) = \sum_{p\in\mathbb{Z}}(f_p^- e^{-i\beta_{1p}z} + f_p^+ e^{i\beta_{1p}z})e^{i\alpha_p x} + \sum_{p\in\mathbb{Z}}\sum_{l\in\mathbb{Z}} B_l K_{pl}^{\pm} e^{i(\alpha_p x \pm \beta_{1p}z)}. \tag{2}$$

In the above, the $f_p^{\pm}$ coefficients account for the field diffracted by the plate, signs $+$ and $-$ corresponding to waves propagating into the $+z$ and $-z$ directions, respectively. The B_l coefficients are those of the field scattered by the cylinder of the unit cell, and from somewhat standard analysis $K_{pl}^{\pm} = \frac{2(-i)^l e^{\pm il\theta_p}}{d\beta_{1p}}$. Applying the boundary conditions on Γ_a and Γ_b with the field representation in region $R_0^{\pm}$ and R_1, we get the solution for R_p and T_p as

$$\begin{cases} T_p = \frac{1}{D_p}\left[4h_p E_{inc}\delta_{p0} + \sum_{l\in\mathbb{Z}}\frac{8(-i)^l h_p B_l}{d\beta_{1p}}[i\sin(l\theta_p + \beta_{1p}a) - h_p\cos(l\theta_p + \beta_{1p}a)]\right] \\ R_p = -\frac{1}{D_p}\left[2i\sin(\beta_{1p}L)(h_p^2 - 1)E_{inc}\delta_{p0} + \sum_{l\in\mathbb{Z}}\frac{8(-i)^l h_p B_l}{d\beta_{1p}}(h_p\cos(l\theta_p + \beta_{1p}b) + i\sin(l\theta_p + \beta_{1p}b))\right]; \end{cases} \tag{3}$$

here, $D_p = 2i\sin(\beta_{1p}L)(h_p^2 + 1) - 4h_p\cos(\beta_{1p}L)$, $h_p = \frac{\mu_0\beta_{1p}}{\mu_1\beta_{0p}}$. It is obvious that transmission and reflection coefficients are related to the coefficients B_l. They are calculated by the Multipole Method [4, 5]: the field in the vicinity of one cylinder (in an annular domain not intersecting any other cylinder) is expanded into terms of Bessel and Hankel functions as is usual. By comparing the field expansion with the cylindrical form of equation (2) and using rather basic knowledge on cylinders' scattering [6], we derive an iterative relationship that is yielding the sought B_l.

In the study of the diffraction problems related to the periodic structure depicted in Figure 1, *lattice sums* (a class of Schlömilch series) arise naturally. In accord with the fact that we deal with 1-D periodicity in a 2-D scattering case, they are defined as [5] $S_l = \sum_{n=1}^{+\infty} H_l^{(1)}(k_1 nd)[e^{i\alpha_0 nd} + (-1)^l e^{-i\alpha_0 nd}]$ wherein $H_l^{(1)}(x)$ is the first kind Hankel function of l^{th} order, $\alpha_0 = k_0 sin(\theta^i)$. In general, such a representation of the lattice sum is too slowly convergent either for numerical computations or for the determination of the explicit dependence of S_l upon parameters l, d and α_0. Its calculation is key to the efficient numerical solution of B_l, especially when the material of the plate (in region R_1) is lossless (k_1 real-valued). In order to compute it quickly and accurately, an alternative representation is required. Much literature exists on this problem [7, 8], and it is still being actively studied, e.g., [9, 10].

4 On-going work

We have been implementing the above into numerical simulations. Reproduction of results in acoustics [2] in both TE and TM cases via equivalences of density and compressibility to electromagnetic parameters has been reached, underlining that attenuation in the matrix material is much simplifiying the computations.

Yet, in the context of NdT, the computations are expected to complicate. For carbon-fiber-reinforced composites, the (epoxy) matrix exhibits relative permittivities close to 4 (this value is a matter of further discussion) and shows no loss (conductivity below 10^{-10} S/m) while the (carbon) fibers are of conductivities typically about 10^5 S/m, see [11], i.e., we have high electromagnetic contrast vs. the matrix yet no impenetrability at the usual $1 - 10$ MHz range of testing. For glass-fiber-reinforced composites, low permittivity contrasts are observed, with glass fibers of relative permittivities about 6 (varying with frequency) and small but not negligible loss everywhere (imaginary parts of relative permittivities about 0.1 to 0.2) at the usual $1 - 10$ GHz range of testing. To achieve proper convergence of the numerical results then remains an issue, since neither the above, well-documented acoustic case nor cases found in the vast literature on photonic crystals apply straightforwardly. It remains also to be seen, provided that the frequency of interrogation is low enough, whether considering the $p = 0$ mode under normal illumination leads to effective ("large-scale") parameters of the panel.

As for advances in the formulation itself, one next step could be on magnetic/electric line sources parallel with the cylinders' axes outside or inside the panel (then, either in the cylinders or outside them) to get the scalar Green's functions of the undamaged structure. But it seems to be more interesting to directly extend what has been done so far to a 2.5-dimensional case when the plane of incidence of the plane waves is no more orthogonal to the cylinders' axes (TE and TM couple),with aiming at constructing the dyadic Green's functions of interest. Ultimately, the analysis must develop onto the damaged case (one or several missing cylinders) via a properly designed field integral formulation.

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